Power Scaling in Port-Hamiltonian Based Bilateral Telemanipulation

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Abstract—In several applications involving bilateral telemanipulation, master and slave robots act at different power scales (e.g. telesurgery). The aim of this paper is to embed power scaling into port-Hamiltonian based bilateral telemanipulation schemes. In order to deal with non negligible transmission delays we propose a novel scattering based communication strategy to properly scale the power exchanged by master and slave while preserving a stable behavior of the overall scheme.

Index Terms—Telemanipulation, power scaling, port-Hamiltonian systems

I. INTRODUCTION

A bilateral telemanipulation system is basically made up of three parts: a controlled local robot (master side), a controlled remote robot (slave side) and a medium, usually characterized by a non-negligible delay, through which master and slave sides are interconnected and exchange information (communication channel). Stability is a key issue in the implementation of a bilateral telemanipulation system since both the non-negligible time delay in the communication between master and slave sides and the interaction with unknown environments can destabilize the whole system.

Passivity is a very suitable tool to stabilize a telemanipulator; in fact, implementing each part of a telerobotic system as a passive system and interconnecting them in a power preserving way it is possible to achieve an intrinsically passive system which is consequently characterized by a stable behavior.

In [1], [9] scattering theory has been used to build a communication channel that is passive independently of any constant delay. In [10], discrete scattering is used to implement a switching packet transmission line that is passive also when communication delay is variable and some packets are lost. Several passivity based strategies have been proposed to passively control master and slave sides. In particular, in [12], a generic framework for geometric telemanipulation of port-Hamiltonian systems ([13]) has been proposed: the passively controlled master and slave sides are modeled as port-Hamiltonian systems which are interconnected through a scattering based communication channel. For an overview and a comparison of the various techniques proposed in the literature see, for instance, [3].

In several tasks involving bilateral telemanipulators, such as telesurgery and telemanipulation of huge robotic arms for extra-vehicular activity in space applications, master and slave act at different power scales and therefore, it is necessary to scale velocities and forces that are exchanged. Ideally, the system should transmit to the human operator a scaled version of the environment impedance such that the dynamic character of the environment remains undistorted.

Several researchers addressed this problem and several control algorithms for achieving velocity/force scaling have been proposed. In [11], [4] linear teleoperators have been considered; in [11] loop-shaping compensators have been introduced in a position/force architecture to increase transparency of the overall telemanipulation scheme. In [4] \( H_\infty \) control and \( \mu \)-synthesis have been used to implement power scaling over the Internet. In [7] robots characterized by a constant inertia tensor have been considered and a control law based on the cancellation of dynamics has been suggested. In [8] the telesmanipulator is decomposed in a shape and locked system and a passive feed-forward action is used to implement a scaled coordination between master and slave robots.

Scaling forces and velocities means scaling the power exchanged between master and slave sides. Thus, it is natural to assign the issue of power scaling to the medium through which master and slave side exchange power, namely the communication channel. In [2], [5] linear telemanipulators are considered and the problem of scaling is tackled from this point of view. In [2] it is shown that a scaling of the exchanged power can be performed without affecting passivity of the overall scheme and in [5] this concept is further analyzed and a necessary condition for the stability in case of multichannel amplification is provided. However, in both cases, negligible communication delay has been assumed.

In this paper we will consider the intrinsically passive port-Hamiltonian based bilateral telemanipulation scheme proposed in [12], that allows considering generic non linear robots, and we will show how to embed power scaling in the interconnection between master and slave sides. We will firstly prove that, analogously to what has been done in [2] for linear telesmanipulators, in case of negligible communication delay, power scaling can be achieved without compromising the dissipative, and, therefore, stable, behavior of the
telemanipulation scheme. Secondly, we will propose a novel scattering based communication strategy that allows to safely scale the power exchanged also in case of non negligible transmission delay.

The paper is organized as follows: in Sec. II some background on intrinsically passive bilateral telemanipulation for port-Hamiltonian systems is given and in Sec. III the problem of power scaling in case of negligible communication delay is addressed. In Sec. IV the problem of power scaling is solved in case of non-negligible communication delay and in Sec. V some simulations are proposed in order to validate the results of the paper. Finally, in Sec. VI some conclusions are drawn and some future work is addressed.

II. BACKGROUND

A. Port-Hamiltonian systems

We will now try to give an intuitive description of port-Hamiltonian systems using coordinates in order to concentrate on the prime contribution of the paper. More formal descriptions can be found in [13]. We can consider a port-Hamiltonian system as composed of a state manifold $X$, a lower bounded energy function $H : X \rightarrow \mathbb{R}$ corresponding to the internal energy, a network structure $D(x) = -D^T(x)$ whose graph has the mathematical structure of a Dirac structure ([6]), which is in general a state dependent power continuous interconnection structure, and an interconnection port represented by a pair of dual power variables $(e, f) \in V^* \times V$ called effort and flow respectively. This port is used to interact energetically with the system: the power supplied through a port is equal to $e(f)$ or, using coordinates, to $e^T f$. We can furthermore split the interaction port in more subports, each of which can be used to model different power flows. We will indicate with the subscript $I$ the power ports by means of which the system interacts with the rest of the world, with the subscript $C$ the power ports associated with the storage of energy and with the subscript $R$ the power ports relative to power dissipation.

The dissipation in the system can be modeled using as characteristic equations $e_R = R(x)f_R$ with $R(x)$ a symmetric and positive semi-definite matrix. If we furthermore set $\dot{x} = f_C$ and $e_C = \frac{\partial H}{\partial x}$, due to the skew-symmetry of $D(x)$, we obtain the following power balance:

$$\dot{H} + f_R^T R(x) f_R = e_C^T f_I$$

(1)

which clearly says that the supplied power $e_C^T f_I$ equals the increase of internal energy plus the dissipated one and that, therefore, a port-Hamiltonian system is passive.

A very broad class of physical systems, both linear and non-linear, can be modeled within the port-Hamiltonian framework which can therefore be used to model telemanipulation systems endowed with nonlinear robots.

B. Port-Hamiltonian based bilateral telemanipulation

The aim of this subsection is to give some background on bilateral telemanipulation based on port-Hamiltonian systems; for further details the reader is addressed to [12], [10]. The port-Hamiltonian based bilateral telemanipulation scheme is represented in Fig. 1 in a bond-graph notation. The robot (either master or slave) can be modeled as a port-Hamiltonian system and it is interconnected in a power preserving way to a port-Hamiltonian controller which acts as an intrinsically passive impedance controller (IPC) as reported in [12]. Master and slave sides exchange power through a transmission line that is characterized, in general, by a non negligible transmission delay.

Each port boundary of which master and slave sides exchange power through the communication channel is characterized by an effort $e(t)$ and by a flow $f(t)$. The power flowing into the channel is given by $e^T(t) f(t)$ and it can be decomposed into an incoming power wave $s^+(t)$ and an outgoing power wave $s^-(t)$ in such a way that

$$e^T(t) f(t) = \frac{1}{2} ||s^+(t)||^2 - \frac{1}{2} ||s^-(t)||^2$$

(2)

where $|| \cdot ||$ is the standard Euclidean norm and

$$s^+(t) = \frac{1}{\sqrt{2}} N^{-1}(e(t) + Z f(t))$$

$$s^-(t) = \frac{1}{\sqrt{2}} N^{-1}(e(t) - Z f(t))$$

(3)

where $Z = NN > 0$ is the positive definite impedance of the scattering transformation. In order to get a passive exchange of energy independently of any constant communication delay, the power ports connected to the transmission line are decomposed into a pair of scattering variables which are transmitted along the channel. In particular, the following communication strategy is implemented

$$s^+_{m}(t) = s^+_s(t - T)$$

$$s^+_{s}(t) = s^+_m(t - T)$$

$$s^-_{m}(t) = s^-_s(t - T)$$

$$s^-_{s}(t) = s^-_m(t - T)$$

(4)

where the pedices $m$ and $s$ indicate master and slave side respectively and $T$ represents the transmission delay. Using this strategy, the communication channel is a lossless system and the power wave outgoing from the master side becomes the incoming power wave at the slave side and viceversa. The system will read the incoming scattering wave $s^+_s(t)$ and output power variable (either the effort or the flow) and will calculate the input power variable (either the flow or the effort) and the scattering wave $s^-_s(t)$ to transmit through the communication channel.
Since a port-Hamiltonian based bilateral telemanipulator is made up of passive subsystems interconnected in a power preserving way, the overall system is intrinsically passive and, therefore, characterized by a stable behavior.

III. POWER SCALING IN CASE OF NEGLIGIBLE COMMUNICATION DELAY

In this section we consider port-Hamiltonian based bilateral telemanipulators where the communication channel is characterized by a negligible delay. We will prove, generalizing what has been proven in [2] for linear teleoperators, that it is possible to implement power scaling while preserving a dissipative, and therefore stable, behavior of the overall scheme.

Since the power preserving interconnection of two port-Hamiltonian systems is again a port-Hamiltonian system [13], we will model master and slave sides (robot + controller) as two port-Hamiltonian systems characterized by energy functions and states $H_m, x_m$ and $H_s, x_s$ respectively. The power ports through which master and slave sides are interconnected to the scattering based communication channel, will be denoted by $(e_m, f_m)$ and $(e_s, f_s)$ respectively. In case of negligible delay and if master and slave side have different causalities at the ports $(e_m, f_m)$ and $(e_s, f_s)$, it is possible to implement the interconnection between master and slave side using directly effort and flow variables.

Remark 1: In case master and slave side have the same causality at the ports $(e_m, f_m)$ and $(e_s, f_s)$, some communication delay is necessary [12] for implementing the interconnection. Thus, in order to implement power scaling in this case, we refer to next section.

The considered telemanipulation scheme is reported in Fig. 2 where the power ports $(e_H, f_H)$ and $(e_E, f_E)$ represent the means through which the human operator interacts with the master robot and the remote environment interacts with the slave robot respectively. In case of non scaled telemanipulation, master and slave sides are joined through a power preserving interconnection, namely an interconnection that allows just an exchange of power without any amplification/attenuation. There are infinite possible power preserving interconnections but one of the most used in bilateral telemanipulation is the common effort interconnection which is described by

$$
\begin{align*}
    e_s(t) &= e_m(t) \\
    f_s(t) &= -f_m(t)
\end{align*}
$$

Using this interconnection strategy the power supplied to the slave side by means of $(e_s(t), f_s(t))$ is exactly that extracted from the master side through $(e_m(t), f_m(t))$ since:

$$
P_s(t) = e_s^T(t) f_s(t) = -e_m^T(t) f_m(t) = -P_m(t)
$$

where $P_s(t)$ and $P_m(t)$ indicate the incoming power flow at the master and slave sides respectively. It can be easily proven, exploiting the passivity of port-Hamiltonian systems, that the overall scheme is passive, [13]. In order to allow a scaling on the interconnecting ports and, consequently, on the power exchanged between master and slave sides we propose to use the following power scaled common effort interconnection:

$$
\begin{align*}
    e_s(t) &= \alpha e_m(t) \\
    f_s(t) &= -\beta f_m(t)
\end{align*}
$$

where $\alpha$ and $\beta$ are the scaling factors. The scaling factors for the efforts and for the flows can be different since it can be desirable to differently scale the motions and the exchanged forces.

The following result can be proven:

Proposition 1: If the interconnection between master and slave is made through a power scaled common effort interconnection, then the overall systems is dissipative.

Proof: Since master and slave sides are port-Hamiltonian systems, then

$$
\begin{align*}
    e_H^T(t) f_H(t) + e_m^T(t) f_m(t) &= \dot{H}_m(t) + P_{dm}(t) \\
    e_E^T(t) f_E(t) + e_s^T(t) f_s(t) &= \dot{H}_s(t) + P_{ds}(t)
\end{align*}
$$

where $H_m, P_{dm}$ and $H_s, P_{ds}$ are the lower bounded energy function and the nonnegative power dissipated characterizing master and slave sides respectively and it has been set $H_m(t) = H_m(x_m(t))$ and $H_s(t) = H_s(x_s(t))$ to lighten the notation. In case of power scaled common effort interconnection, using Eq.(7), we have that

$$
e_s^T(t) f_s(t) = -\alpha \beta e_m^T(t) f_m(t)$$

Thus, we have that

$$
\begin{align*}
    \alpha \beta e_H^T(t) f_H(t) + \alpha \beta e_m^T(t) f_m(t) + e_s^T(t) f_s(t) + \\
    + e_s^T(t) f_s(t) = \alpha \beta \dot{H}_m(t) + \alpha \beta P_{dm}(t) + \dot{H}_s(t) + \\
    + P_{ds}(t) = \alpha \beta e_H^T(t) f_H(t) + \alpha \beta e_m^T(t) f_m(t) - \\
    - \alpha \beta e_m^T(t) f_m(t) + e_E^T(t) f_E(t) = \\
    = \alpha \beta e_H^T(t) f_H(t) + e_E^T(t) f_E(t)
\end{align*}
$$
and thus
\[
e^{T}_{H}(t)f_{H}(t) + \frac{1}{\alpha \beta}e^{T}_{E}(t)f_{E}(t) =
\]
\[
= H_{m}(t) + \frac{1}{\alpha \beta}H_{s}(t) + P_{dm} + \frac{1}{\alpha \beta}P_{ds}
\]
\[
H(t) P\frac{P_{ds}(t)}{H(t)} \quad (11)
\]

Since both \(H_{m}\) and \(H_{s}\) are lower bounded, also \(H\) is lower bounded and since both \(P_{dm}\) and \(P_{ds}\) are nonnegative also \(P_{d}\) is nonnegative. Therefore, a function of the power supplied to the teleoperator (i.e. \(e^{T}_{H}(t)f_{H}(t) + (1/\alpha \beta)e^{T}_{E}(t)f_{E}(t)\)) is either stored or dissipated, meaning that the overall system is dissipative.

In case of non scaled telemanipulation, \(\alpha = \beta = 1\) and Eq.(11) can be rewritten as
\[
e^{T}_{H}(t)f_{H}(t) = H_{m}(t) + P_{dm} + P_{ds} - e^{T}_{E}(t)f_{E}(t) \quad (12)
\]

Thus, while interacting with the environment, the human operator feels the impedances deriving by the presence of master and slave controlled devices and the exchange of energy with the remote environment. In case of scaled telemanipulation, the interaction of the human operator with the remote environment can be described as
\[
e^{T}_{H}(t)f_{H}(t) =
\]
\[
= H_{m}(t) + P_{dm} + \frac{1}{\alpha \beta}(H_{s}(t) + P_{ds} - e^{T}_{E}(t)f_{E}(t)) \quad (13)
\]

In this case, the human operator perceives the impedance of the master side and a scaled version of the slave side and of the environment impedance. Thus, the effect of the power scaled common effort interconnection is to transmit a scaled version of the slave side to the master side as ideally should be as reported in [11].

**Remark 2:** The scaled interconnection between master and slave prevents from proving passivity of the overall system and allows only proving dissipativity. Dissipativity comes from the fact that it is necessary to scale the energetic behavior of the slave side in order to give to the operator a realistic feeling and thus, the power balance of the overall system has to contain a scaled version of the power exchanged with the environment instead of the “real” power. Nevertheless the fact that the telemanipulation system is dissipative with respect to the supply rate \(e^{T}_{H}(t)f_{H}(t) + (1/\alpha \beta)e^{T}_{E}(t)f_{E}(t)\) is still sufficient for guaranteeing that the overall system is characterized by a stable behavior when interacting with any passive environment. In fact, as it happens in non scaled telemanipulation, \(-e^{T}_{E}(t)f_{E}(t) = H_{e}(t) + P_{de}(t)\), where \(H_{e}(t)\) and \(P_{de}(t)\) are the lower bounded storage function and the non negative dissipation function characterizing the passive environment. Replacing this power balance in Eq.(13) we can see that the overall system is passive and, therefore, characterized by a stable behavior.

The scaling of the power variables gives a consequent scaling of the power exchanged between master and slave as expressed by Eq.(9). The power extracted from the master side is scaled by a factor \(\alpha \beta\) and is supplied to the slave side. Thus, while common effort interconnection is power preserving, namely it simply allows a transfer of power, the power scaled common effort interconnection in general changes the amount of power that is transferred. Therefore, the scaled interconnection is NOT a passive element but, nevertheless, as shown in Proposition 1, it can be safely used in telemanipulation. This happens because the effect of the scaling is to “mask” the slave side and to make it appear to the human operator as if it acted at the same power scale of the master side. However, this amplification/attenuation of the power transmitted does NOT modify the kind of dynamic behavior of the slave side which keeps on being passive. As a result we have two systems characterized by a passive behavior but acting at different power scales and this leads to a behavior which is passive with respect to the sum of the power injected at one side and the scaled power injected at the other side.

**IV. POWER SCALING IN CASE OF NON-NEGLECTIBLE COMMUNICATION DELAY**

In non scaled telemanipulation, scattering theory is used to make the energetic interconnection between master and slave robust, in the sense that it preserves passivity, with respect to communication delay.

In fact, it has been proven in [9] that when using the common effort interconnection reported in Eq.(5) in the presence of non negligible transmission delay, the communication channel becomes a non passive element and the overall telemanipulation scheme is consequently characterized by an unstable behavior. If the interconnection is made through scattering variables, passivity of the overall system is preserved independently of any communication delay. In this section we want to exploit scattering variables to implement the power scaled common effort interconnection over a delayed transmission line.

In order to find the scattering based communication strategy for power scaling interconnections, we proceed as in the case of power preserving interconnections. First we find the equivalent expression, in terms of scattering variables, of the power scaled common effort interconnection reported in Eq.(7); secondly we analyze the behavior of the overall telemanipulation scheme when the energetic interconnection between master and slave is made through the scattering based power scaled common effort interconnection and the communication channel is characterized by a non negligible delay.

Consider the power scaled common effort interconnection described by Eq.(7). Given a power port \((e(t), f(t))\), the following relations can be easily derived from Eq.(3):
\[
\left\{
\begin{array}{l}
  e(t) = \frac{N}{N_{s}}(s^{+}(t) + s^{-}(t)) \\
  f(t) = \frac{1}{N_{s}}N^{-1}(s^{-}(t) - s^{+}(t))
\end{array}
\right.
\quad (14)
\]

Using Eq.(14) with Eq.(7) we have that the scattering based expression of the power scaled common effort interconnection
is given by:
\[
\begin{align*}
    s^+_{1}(t) + s^-_{1}(t) &= \alpha(s^+_{m}(t) + s^-_{m}(t)) \\
    s^+_{2}(t) - s^-_{2}(t) &= -\beta(s^+_{m}(t) - s^-_{m}(t))
\end{align*}
\]  
(15)

where the pedices \( m \) and \( s \) indicate master and slave side respectively. After some simple computations we have that Eq.\((15)\) can be rewritten as:
\[
\begin{align*}
    s^+_{1}(t) &= \frac{2\alpha\beta}{\alpha + \beta}s^-_{m}(t) + \frac{\alpha - \beta}{\alpha + \beta} s^+_{s}(t) \\
    s^+_{m}(t) &= \frac{\beta - \alpha}{\alpha + \beta} s^-_{m}(t) + \frac{2}{\alpha + \beta} s^+_{s}(t)
\end{align*}
\]  
(16)

Consider now that a communication delay \( T \) is present in the interconnection between master and slave. The scattering based communication strategy can be obtained by Eq.\((16)\) and is given by:
\[
\begin{align*}
    s^+_{1}(t) &= \frac{2\alpha\beta}{\alpha + \beta}s^-_{m}(t - T) + \frac{\alpha - \beta}{\alpha + \beta} s^+_{s}(t - T) \\
    s^+_{m}(t) &= \frac{\beta - \alpha}{\alpha + \beta} s^-_{m}(t - T) + \frac{2}{\alpha + \beta} s^+_{s}(t - T)
\end{align*}
\]  
(17)

**Remark 3:** Consider the master side and the incoming scattering wave \( s^-_{m}(t - T) \). The term \( s^+_{s}(t - T) \) arrives from the communication channel while the term \( s^-_{m}(t - T) \) has to be added at the master side; this can be done by implementing a buffering strategy that stores a copy of the outgoing wave \( s^+_{m}(t) \) for a time interval of length \( T \). In case the communication delay is unknown, the delay can be recovered by attaching a time stamp to the transmitted wave. Similar considerations hold for the slave side.

Consider the telemanipulation scheme reported in Fig. 2. The following result holds:

**Proposition 2:** Suppose that the interconnection between master and slave sides is given by Eq.\((17)\) where \( T \) is the communication delay. Then the overall telemanipulation system is dissipative.

**Proof:** Using Eq.\((17)\) and recalling that the standard Euclidean norm is used for scattering variables we have that
\[
\begin{align*}
    \frac{1}{2}\alpha\beta ||s^-_{m}(t)||^2 - \frac{1}{2}\alpha\beta ||s^+_{m}(t)||^2 - \frac{1}{2} ||s^+_{s}(t)||^2 -
\end{align*}
\]
\[
\begin{align*}
    -\frac{1}{2} ||s^-_{s}(t)||^2 &= \frac{1}{2}\alpha\beta ||s^+_{m}(t)||^2 - \frac{1}{2} \frac{\alpha\beta s^-_{m}(t) - \alpha\beta s^+_{m}(t - T)||^2 -
\end{align*}
\]
\[
\begin{align*}
    -\frac{1}{2} \frac{4\alpha\beta}{(\alpha + \beta)^2} ||s^-_{s}(t - T)||^2 - \frac{1}{2} \frac{4\alpha\beta}{(\alpha + \beta)^2} ||s^-_{s}(t - T)||^2 -
\end{align*}
\]
\[
\begin{align*}
    +\frac{1}{2} ||s^-_{m}(t - T)||^2 - \frac{1}{2} \frac{4\alpha\beta}{(\alpha + \beta)^2} ||s^-_{m}(t - T)||^2 -
\end{align*}
\]
\[
\begin{align*}
    -\frac{1}{2} \frac{(\alpha - \beta)^2}{(\alpha + \beta)^2} ||s^-_{m}(t - T)||^2 -
\end{align*}
\]
\[
\begin{align*}
    -\frac{1}{2} \frac{4(\alpha - \beta)^2}{(\alpha + \beta)^2} ||s^-_{m}(t - T)||^2 -
\end{align*}
\]
\[
\begin{align*}
    -\frac{1}{2} \frac{4\alpha\beta}{(\alpha + \beta)^2} s^-_{m}(t - T)^T s^-_{s}(t - T) = 
\end{align*}
\]
\[
\begin{align*}
    = \frac{\alpha}{\beta} \left[ \int_{-T}^{t} \frac{1}{2}(\alpha\beta ||s^-_{m}(\tau)||^2 + ||s^-_{s}(\tau)||^2) \, d\tau \right] = H_{ch}
\end{align*}
\]  
(18)

where \( H_{ch} \) is a non negative function. From Eq.\((18)\) we have that
\[
\begin{align*}
    \frac{1}{2} ||s^+_{m}(t)||^2 - \frac{1}{2} ||s^-_{m}(t)||^2 = 
\end{align*}
\]
\[
\begin{align*}
    = -\alpha(\frac{1}{2} ||s^+_{m}(t)||^2 - \frac{1}{2} ||s^-_{m}(t)||^2) - H_{ch}
\end{align*}
\]  
(19)

Using Eq.\((2)\) with Eq.\((19)\) we have that
\[
\begin{align*}
    e_{s}^T(t) f_s(t) = -\alpha e_{s}^T(t) f_s(t) + H_{ch}
\end{align*}
\]  
(20)

Consider now the power flow along the overall telemmanipulation system. Using Eq.\((20)\), referring to Fig. 2, we have that:
\[
\begin{align*}
    \alpha e_{s}^T(t) f_s(t) + \alpha e_{m}^T(t) f_s(t) + e_{s}^T(t) f_s(t) +
\end{align*}
\]
\[
\begin{align*}
    + e_{E}^T(t) f_E(t) = \alpha \beta \dot{H}_{m}(t) + \alpha \beta P_{dm}(t) + \dot{H}_{s}(t) +
\end{align*}
\]
\[
\begin{align*}
    + P_{ds}(t) = \alpha \beta e_{s}^T(t) f_s(t) + \alpha \beta e_{m}^T(t) f_m(t) -
\end{align*}
\]
\[
\begin{align*}
    -\alpha \beta e_{m}^T(t) f_m(t) + \dot{H}_{ch}(t) + e_{E}^T(t) f_E(t)
\end{align*}
\]

Thus
\[
\begin{align*}
    e_{E}^T(t) f_E(t) = 
\end{align*}
\]
\[
\begin{align*}
    \dot{H}_{m}(t) + \frac{1}{\alpha \beta} \dot{H}_{s}(t) + \frac{1}{\alpha \beta} \dot{H}_{ch}(t) +
\end{align*}
\]
\[
\begin{align*}
    + P_{dm}(t) + \frac{1}{\alpha \beta} P_{ds}(t)
\end{align*}
\]  
(21)

Notice that \( P_{d} \) is a non negative function since it is the sum of two non negative functions and \( H \) is a lower bounded function since it is the sum of three lower bounded functions. Thus, a function of the power supplied to the teleoperator (i.e. \( e_{E}^T(t) f_E(t) + (1/\alpha \beta) e_{s}^T(t) f_E(t) \)) is either stored or dissipated, meaning that the overall system is dissipative.

**Remark 4:** In case of \( \alpha = \beta \) we have that the communication strategy is simplified and it results
\[
\begin{align*}
    s^+_{1}(t) &= \alpha s^-_{m}(t - T) \\
    s^+_{m}(t) &= \frac{1}{\alpha} s^-_{s}(t)
\end{align*}
\]  
(22)

In this case no buffering is needed for the implementation of the interconnection.

**Remark 5:** In case \( \alpha = \beta = 1 \), namely in case of no scaling, we recover the communication strategy that is usually exploited in telemanipulation, namely
\[
\begin{align*}
    s^+_{1}(t) &= s^-_{m}(t - T) \\
    s^+_{m}(t) &= s^-_{s}(t - T)
\end{align*}
\]  
(24)

Thus, the communication strategy proposed in Eq.\((17)\) represents a generalization what is commonly used in passivity based bilateral telemanipulation.
The simulation results are reported in Fig. 3. In Fig. 3(b) the power flows at the master and slave sides are reported. We can see that the power that is extracted from the master side is, as expected, amplified by a factor $\alpha \beta = 6$ and supplied to the slave side. Thus we can see that we inject into the remote side an amount of power greater than what we extracted from the master side and that therefore the communication channel acts as a non passive power amplifier. Nevertheless the behavior of the system is stable, as expected from Proposition 2. In Fig. 3(c) the flows exchanged between master and slave sides are reported. Since we are implementing a power scaled common effort interconnection we have that the flows have discording signs, however, because of the scaling, we have that the absolute value of $f_\alpha$ is three times bigger than $f_m$. Finally, in Fig. 3(d), the effort exchanged are reported. We can see that the effort at the slave side is twice bigger than that at the master side and, therefore, the scaling is respected in the interconnection.

In the next simulation, a contact task is implemented. The operator applies a constant force to the master and the slave interacts with a rigid wall, implemented as the parallel of a stiff spring, $k = 1000 \ N/m$, and a damper, $b = 100 \ N/sec/m$, set at the position $x = 0.5 \ m$. The communication delay is $T = 0.5 \ sec$, and the scaling factors are $\alpha = \beta = 2$. The results of the simulation are reported in Fig. 4. In Fig. 4(a) we can see the positions of master and slave. The slave stops when it meets the wall. The interaction force is scaled and fed back to counteract the force applied by the operator on the master robot which, therefore, stops as well. In Fig. 4(b) we can see that the power extracted from the master side is amplified by a factor $\alpha \beta = 4$ and supplied to the slave side and in Fig. 4(c) and Fig. 4(d) the flows and efforts exchanged are reported. Notice that the effort at the slave side is twice the position of the slave is three times bigger than that of the master as expected. In Fig. 3(b) the power flows at the master and slave sides are reported. We can see that the power that is extracted from the master side is, as expected, amplified by a factor $\alpha \beta = 6$ and supplied to the slave side. Thus we can see that we inject into the remote side an amount of power greater than what we extracted from the master side and that therefore the communication channel acts as a non passive power amplifier. Nevertheless the behavior of the system is stable, as expected from Proposition 2. In Fig. 3(c) the flows exchanged between master and slave sides are reported. Since we are implementing a power scaled common effort interconnection we have that the flows have discording signs, however, because of the scaling, we have that the absolute value of $f_\alpha$ is three times bigger than $f_m$. Finally, in Fig. 3(d), the effort exchanged are reported. We can see that the effort at the slave side is twice bigger than that at the master side and, therefore, the scaling is respected in the interconnection.

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bigger than the effort at the slave side because of the scaling imposed. Thus, the operator perceives a scaled version of the interaction of the slave with the remote environment. From this simulation we can see that the scattering based power scaled common effort interconnection transmits to the master side a scaled version of the slave side and, nonetheless power scaling is NOT a passive operation, a stable behavior of the overall system is achieved.

VI. CONCLUSIONS AND FUTURE WORK

In this work we have shown how to implement power scaling in port-Hamiltonian based bilateral telemanipulation. We have illustrated that it is possible to embed power scaling in the interconnection between master and slave sides, which are modeled as port-Hamiltonian systems, while preserving a dissipative behavior of the overall telemanipulation scheme. Then, we have used scattering theory to make robust the power scaled interconnection with respect to any constant communication delay and we have obtained a novel scattering based interconnection strategy that allows power scaling while preserving a dissipative behavior of the system. In this way we have transformed the scattering based communication channel from a fixed entity, characterized by the fixed equations reported in Eq.(4), into a tunable entity characterized by Eq.(17) and by two parameters \( \alpha \) and \( \beta \) that can be changed in order to provide a desired scaling. In this way, all the issues relative to power scaling are assigned to the interconnection. If we want to implement a power scaled bilateral telemanipulation scheme, we just need to take the master and slave robots with their (possibly embedded) control system and to interconnect them through a scattering based communication channel that has to be tuned in order to meet the different power scales at which the devices operate.

Future work will deal with the extension of power scaling to packet switched communication channels and with the analysis of the effects of packets loss and of variable delay. Furthermore, the possibility to achieve a variable power scaling will be investigated. Finally, we want to extend the proposed interconnection strategy in order to be able to scale with different factors the components of flows and efforts exchanged between master and slave side.

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