Delayed Virtual Environments: a port-Hamiltonian Approach

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Abstract—In this paper the problem of delayed virtual environments in haptics is addressed. We show that the approach outlined in [7] is no longer passive in case of (computational) delay on the output of the virtual environment. Passivity can be recovered using scattering theory; a discretization algorithm which leads to a discrete passive port-Hamiltonian systems with respect to any delay on the output is proposed.

I. INTRODUCTION

An haptic display is basically a system which allows a human operator to interact, by means of some robotic interface, with a simulated virtual environment. Passivity theory is a very suitable tool for the study of interaction tasks: it is sufficient for a system to be passive in order to have a stable behavior both interacting and non interacting with other (passive) systems [6]. Furthermore, Hogan [3] proved that the human operator behaves as a passive system in the frequency range of interest in haptics.

Several researches have been carried on this direction and several algorithms have been proposed. Colgate has introduced the idea of virtual coupling, namely a virtual layer between the virtual environment and the haptic device which makes possible to guarantee passivity for arbitrary passive virtual environment and even for a class of non passive ones ([5]). Hannaford noted that a fixed parameters virtual layer can decrease performances in some working conditions and he introduced a variable virtual layer which is, loosely speaking, a virtual damper which is activated only when some energy production is detected (PO/PC strategy, [2]).

An haptic interface can be modeled as the energetic interconnection of four systems: the human operator, the haptic device, the Sample & Hold device and the discrete virtual environment. Both the human operator and the haptic device are passive systems while some extra energy injection can derive both from the Sample & Hold device and from the discrete virtual environment.

In [7] a new approach for the implementation of an haptic interface has been proposed. Instead of adding an extra layer to dissipate the energy produced, the authors have implemented in a passive way each possible source of energy. A Sample & Hold algorithm that allows to inter-connect continuous and discrete systems without injecting extra energy has been introduced. Furthermore, in order to be able to represent any passive physical system, port-Hamiltonian formalism ([9]) has been adopted to represent virtual environments and a new discretization algorithm preserving passivity of port-Hamiltonian systems in their discrete counterpart has been introduced. Finally, interconnecting in a power consistent way ([6]) each component, an intrinsically passive haptic interface has been obtained.

The advantage of this scheme over other methods is that passivity is preserved independently of the sampling period and that any passive dynamics that can be modeled by a port-Hamiltonian system can be discretized and safely used as a virtual environment.

It can happen that the virtual environment dynamics are very complex and that the computation time needed to evolve its simulation exceed the sample period. In these cases we speak of delayed virtual environment. Several researchers addressed this problem and various methodologies have been proposed; in [4], for instance, an extension of the virtual coupling technique, based on input and output strict passivity has been proposed.

The aim of this paper is to extend the scheme proposed in [7] in order to keep into account possibly delayed environments. The paper is organized as follows: in Sec.II we will shortly review the port-Hamiltonian formalism and the scheme proposed in [7], in Sec.III we will introduce the delayed output problem for port-Hamiltonian systems and we will show how to use scattering theory to deal with it. In Sec.IV we will generalize the discretization algorithm proposed in [7] in order to obtain a passive discrete port-Hamiltonian system even in case of delay on the output and in Sec.V some simulations are shown in order to validate our results. Finally, in Sec.VI some conclusions and future work will be addressed.

II. BACKGROUND

A. Continuous port-Hamiltonian systems

The aim of this subsection is to provide some background on port-Hamiltonian systems; more formal descriptions can be found in [9]. We can consider a port-Hamiltonian system as composed of a state manifold \( \mathcal{X} \), an energy...
function $H : \mathcal{X} \to \mathbb{R}$ corresponding to the internal energy, a network structure $D(x) = -D(x)^T$ whose graph has the mathematical structure of a Dirac structure, which is in general a state dependent power continuous interconnection structure, and an interconnection port (power port) represented by an effort-flow pair $(e, f) \in V^* \times V$ which is geometrically characterized by dual vector elements. This port is used to interact energetically with the system. The power supplied through a port is equal to $e^T f$ or using coordinates to $e^T f$. We can furthermore split the interaction port in more sub-ports, each of which can be characterized using coordinates $e^T f$. We can furthermore split the interaction port in more sub-ports, each of which can be characterized using coordinates $e^T f$. We can furthermore split the interaction port in more sub-ports, each of which can be characterized using coordinates $e^T f$. We can furthermore split the interaction port in more sub-ports, each of which can be characterized using coordinates $e^T f$.

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Function $H$ where:

$$H = H_1 + H_2$$

where

$$(\begin{pmatrix} e_1 \\ f_C \\ e_C \\ f_R \end{pmatrix}) = D(x) \begin{pmatrix} f_1 \\ f_C \\ e_C \\ f_R \end{pmatrix}$$

where

$$D(x) := \begin{pmatrix} D_I & G_1 & G_2 \\ -G_1^T & D_C & G_3 \\ -G_2^T & -G_3^T & D_R \end{pmatrix}$$

and $D_I, D_C, D_R$ are skew-symmetric. Due to the skew-symmetry of $D(x)$, we have, using coordinates:

$$P_I + P_C + P_R := e_1^T f_1 + e_C^T f_C + e_R^T f_R = 0 \quad (1)$$

which is a power balance meaning that the total power coming out of the network structure should be always equal to zero.

A dissipating element of the system can be modeled using as characteristic equations $e_R = R(x)f_R$ with $R(x)$ a symmetric and positive semi-definite matrix. This implies that

$$f_R = (D_R - R)^{-1} G_2^T f_I + (D_R - R)^{-1} G_3^T e_C$$

and therefore

$$(\begin{pmatrix} e_1 \\ f_C \end{pmatrix}) = \begin{pmatrix} B & A \\ C & D \end{pmatrix} \begin{pmatrix} f_1 \\ e_C \end{pmatrix}$$

where

$$B := D_I + G_2(D_R - R)^{-1} G_2^T$$

$$A := G_1 + G_2(D_R - R)^{-1} G_3^T$$

$$C := -G_1^T + G_3(D_R - R)^{-1} G_2^T$$

$$D := D_C + G_3(D_R - R)^{-1} G_3^T$$

If we furthermore set $x = f_C$ and $e_C = \frac{\partial H}{\partial x}$, due to the previous power balance we obtain:

$$\dot{H} + f_R^T R(x) f_R = -e_1^T f_1 \quad (7)$$

which clearly says that the supplied power $-e_1^T f_1$ equals the increase of internal energy $\dot{H}$ plus the dissipated one.

### B. Discrete port-Hamiltonian systems

Hereafter we will briefly review how to discretise a port-Hamiltonian system preserving its passivity. We can describe a discrete time port-Hamiltonian system as a continuous time port-Hamiltonian system in which the port variables are frozen for a sample interval $T$. In what follows we indicate with $v(k)$ the value of the discrete variable $v(t)$ corresponding to the interval $t \in [kT, (k + 1)T]$. If we rewrite Eq.(1) for the discrete case, we have:

$$e_I^T f_I(k) + e_C^T f_C(k) + e_R^T f_R(k) = 0 \quad (8)$$

Furthermore, during the interval $k$, we have to consider a constant state $x(k)$ corresponding to the continuous time state $x(t)$. This implies that during the interval $k$, the dissipated energy will be equal to $T f_R^T(k) R(x(k)) f_R(k)$ and the supplied energy will be equal to $-T f_1^T(k) f_1(k)$. In order to be consistent with the energy flows, and as a consequence conserve passivity, we need therefore a jump in internal energy $\Delta H(k)$ from instant $kT$ to instant $(k + 1)T$ such that:

$$\Delta H(k) = -T f_1^T(k) R(x(k)) f_R(k) - T e_1^T(k) f_1(k)$$

This implies that the new discrete state $x(k + 1)$ should belong to an energetic level such that:

$$H(x(k + 1)) = H(x(k) + \Delta H(k))$$

Solving the previous equations in $x(k + 1)$ it is possible to find a set of states the system can jump to preserving passivity. This set can be either empty or have more solutions. In the first case a state is chosen by means of the so-called energy leap strategy while in the second case we have to choose the “closest” (the definition of closeness can be made clear by specifying a proper affine connection on the state manifold) state to the current one. For further details the reader is referred to [7].

### C. Passive interconnection

Consider the port interconnection of a continuous time Hamiltonian system $H_C$ and a discrete Hamiltonian system $H_D$ through a sampler and zero-order hold as shown in Fig.1. If the Sample & Hold is not properly designed, it can happen that the process generates extra energy and that the passivity of the whole system is not assured even if the two interconnected systems are passive.
Suppose that $H_C$ has an admittance causality (effort in/flow out) and therefore $H_D$ has an impedance causality (flow in/effort out). We will have that:

$$e(t) = e_d(k), \quad t \in [kT, (k + 1)T]$$

The following theorem can be proven [7]:

**Theorem 1 (Sample Data passivity):** If we define for the interconnection port of $H_D$

$$f_d(k) := \frac{x(kT) - x((k + 1)T)}{T}, \quad (9)$$

where $x()$ represents the integral of the continuous flow, we obtain an equivalence between the continuous time and discrete time energy flow in the sense that for each $n$:

$$\sum_{i=0}^{n-1} e_d^T(i) f_d(i) T = -\int_0^{nT} e^T(s) f(s) ds \quad (10)$$

From the previous considerations, it is possible to understand that at each sampling time, we have an EXACT matching between the physical energy going into the continuous time system and the virtual energy coming from the discrete time port independently of the sample time $T$ and of eventual intersample dynamics of the continuous system. It is remarkable that the choice reported in Eq.(9) which is very simple and at the same time attractive due to the fact that it corresponds to position measurements, in practice gives such a powerful and at the same trivial result.

The only energy leakage is due to the fact that the discrete time system has no way what so ever to predict the value of the continuous time system at the interconnection port and this implies that only at the end of the sample period will have an exact measure of the energy it supplied to the continuous time system. But the amount of the eventually produced extra energy is exactly known and it can be compensated with a damping circuit or by a clever bookkeeping strategy.

### III. DELAYED OUTPUT

The aim of this section is to analyze the effect of a delay on the output on the passivity of a port-Hamiltonian system.

**Definition 1:** A system is passive if the supplied power is either stored (with a lower bounded storage function) or dissipated.

It follows directly from the definition that a system is passive if:

$$P = \frac{dE}{dt} + P_{\text{diss}}$$

where $P$ represents the supplied power, $E$ a lower bounded function representing the stored energy and $P_{\text{diss}} \geq 0$ the dissipated power. If $P_{\text{diss}} < 0$, it means that there is production of some extra energy and that, therefore, the system is not passive. A port-Hamiltonian system represented by Eq.(2) is passive. In fact the supplied power is $P = -e_1^T f_1$ and passivity directly follows from Eq.(7).

#### A. Effort/Flow Representation

Let us consider a port-Hamiltonian represented by Eq.(2) where the power port by means of which the system energetically interacts with the rest of the world is represented by an effort/flow pair. Assume, furthermore, that the system has impedance causality (i.e. flow in/effort out). In case of delay we have that the output power variable at the interaction port is:

$$e_{1\beta}(t) = e_1(t - \delta)$$

The non delayed system would be passive with respect to the input/output pair $(e_1, f_1)$. It can be easily proved that delay in the output destroys passivity of the port-Hamiltonian. In fact:

$$P(t) = -e_{1\beta}(t) f_1(t) =$$

$$= (e_1(t) + e_{1\beta}(t) - e_1(t))^T f_1(t) = -e_1^T(t) f_1(t) - e_1^T(t) f_1(t) -$$

$$-e_1^T(f(t)) = \frac{dH}{dt} + f_R(l) R(x) f_1(t) - e_1^T(f(t)) f_1(t)$$

Specific choices of the input variable can lead to a negative value of $P_{\text{diss}}$ and therefore to a production of extra energy and to a loss of passivity.

We can therefore conclude that when there is a certain delay in the computation of the output (i.e. in case of delayed virtual environment), port-Hamiltonian systems with an interaction power port represented by an effort/flow pair cannot be safely used because they are not passive.

#### B. Scattering Representation

Scattering variables have been originally introduced in control theory to deal with the problem of communication delay in telemanipulation [1]. Since then, several works have used scattering theory to implement stable telemanipulation systems. In particular a geometric coordinate-free approach to scattering theory has been recently introduced ([8], [9]) and applied to port-Hamiltonian based telemanipulation schemes.

Consider a power port $(e, f)$ where $e$ and $f$ represent an effort and a flow respectively. We can define the scattering waves as:

$$s^+ = \frac{N^{-1}}{\sqrt{2}} (e + Z f) \quad s^- = \frac{N^{-1}}{\sqrt{2}} (e - Z f)$$

where

$$Z = NN$$

is a positive definite matrix representing the impedance of the scattering transformation. The following power balance holds:

$$P = e^T f = \frac{1}{2} ||s^+||^2 - \frac{1}{2} ||s^-||^2 \quad (12)$$
We can, therefore, interpret \( s^+ \) as incoming power and \( s^- \) as outgoing power. A power port represents an exchange of energy between the system and the rest of the world and Eq.(12) shows that this exchange can be equally represented both by an effort/flow pair and by scattering variables.

A definition of passivity based on scattering variables can be given. If \((e, f)\) is the power port by means of which a system interacts with the rest of the world and if \((s^+, s^-)\) is its scattering representation, we have that a system is passive iff \( \exists \beta > 0 \) such that:

\[
\int_0^t \frac{1}{2} \| s^-(\tau) \|^2 d\tau \leq \int_0^t \frac{1}{2} \| s^+ (\tau) \|^2 d\tau + \beta \tag{13}
\]

Loosely speaking, a system is passive iff the outcoming energy bounded by the incoming energy, namely iff there is no internal production of energy.

One could think to deal with delayed virtual environments by simply discretizing a port-Hamiltonian system with power variables as input/output and then to treat the delay in the same way as in telemanipulation. In scattering-based telemanipulation, the power variables of the port of the port-Hamiltonian system are coded into scattering variables and then sent through the communication channel. The port-Hamiltonian system is passive and it has a passive behavior at the power port. By scattering the power port, and transmitting scattering waves, this passive behavior is simply conserved during the communication, independently of any delay.

Unfortunately, when dealing with delayed virtual environments, the power variables at the power port are not consistent and this causes, as shown in Sec.III-A, a non passive behavior of the system at the port. If we used the same strategy used in telemanipulation, the scattering waves would simply replicate the non passive behavior of the systems, leaving the problem unsolved. The delay has to be treated therefore in a different way in case of delayed virtual environment. The reason of this discrepancy is that while in telemanipulation the problem concerning delay is in the transmission of power variables and, therefore, something external to the system. In case of delayed port-Hamiltonian systems the delay is intrinsic into the dynamics of the system, internal to the system, and the scattering framework has to be embedded in the model of the system in order to deal with this kind of delay in a similar way as in [8] for defining impedance matching.

In order to deal with this internal delay we will model a port Hamiltonian system modeling the power port \((e_1, f_1)\), by means of which it interacts with the rest of the world, by the corresponding equivalent scattering representation \((s^+_1, s^-_1)\). Since Eq.(12) holds, we can write

\[
P_l + P_C + P_R = \frac{1}{2} \| s^+_1 \|^2 - \frac{1}{2} \| s^-_1 \|^2 + e_1^T f_C + e_R^T f_R = 0 \tag{14}
\]

which is the power balance in terms of scattered interaction port.

By straightforward calculation we can get the model of a port-Hamiltonian system with scattered interaction port:

\[
\begin{pmatrix}
  s^-_1 \\
  f_C
\end{pmatrix} = \begin{pmatrix}
  S_1 & S_2 \\
  S_3 & S_4
\end{pmatrix} \begin{pmatrix}
  s^+_1 \\
  e_C
\end{pmatrix} \tag{15}
\]

where:

\[
S_1 = (BN^{-1} + N)^{-1} (BN^{-1} - N)^{-1} \tag{16}
\]

\[
S_2 = \sqrt{2}(BN^{-1} + N)^{-1} A \tag{17}
\]

\[
S_3 = \frac{CN^{-1}}{\sqrt{2}} (I - (BN^{-1} + N)^{-1} (BN^{-1} - N)) \tag{18}
\]

\[
S_4 = D - C N^{-1} (BN^{-1} + N)^{-1} A \tag{19}
\]

and now

\[
\dot{H} + f_R^T R(x) f_R = -\left( \frac{1}{2} \| s^+_1 \|^2 - \frac{1}{2} \| s^-_1 \|^2 \right) = P
\]

Let us now consider a port-Hamiltonian system in the form of Eq.(15) and suppose that there is a delay in the computation of the output. We have that:

\[
\begin{cases}
  s^+_{\delta}(t) = 0 & t < \delta \\
  s^+_{\delta}(t) = \tilde{s}^+_I (t - \delta) & t \geq \delta
\end{cases}
\]

where we reasonably assumed that when the output is not yet ready because of the delay, the virtual environment presents 0 on the output buffer.

**Proposition 1:** The delayed scattered port-Hamiltonian system is passive for any delay \( \delta > 0 \).

**Proof:** Since the non delayed system is passive we have that condition (13) holds and therefore that:

\[
\int_0^t \frac{1}{2} \| s^+_{\delta}(\tau) \|^2 d\tau \leq \int_0^t \frac{1}{2} \| s^-_{\delta}(\tau) \|^2 d\tau + \beta \forall t > 0 \tag{20}
\]

Let us now consider the delayed output, \( \bar{s}^+_{\delta}(t) = \tilde{s}^+_I (t - \delta) \). We have that:

\[
\bar{s}^+_{\delta}(t) = 0 \forall t \in [0, \delta]
\]

We can write:

\[
\int_0^t \frac{1}{2} \| s^+_{\delta}(\tau) \|^2 d\tau \leq \int_0^t \frac{1}{2} \| s^-_{\delta}(\tau) \|^2 d\tau \leq \int_0^t \frac{1}{2} \| s_I(\tau) \|^2 d\tau + \beta
\]

which implies:

\[
\int_0^t \frac{1}{2} \| s^-_{\delta}(\tau) \|^2 d\tau \leq \int_0^t \frac{1}{2} \| s^-_{\delta}(\tau) \|^2 d\tau + \beta
\]

Thus the delayed system is passive.
IV. PASSIVE DISCRETIZATION OF PORT-HAMILTONIAN SYSTEMS IN SCATTERING REPRESENTATION

The aim of this section is to modify the passivity preserving discretization algorithm proposed in [7] in order to provide a passive discretization for port-Hamiltonian systems in scattering form. If we rewrite Eq.(14) for the discrete case, we have:

$$\frac{1}{2}\|s_I^+(k)\|^2 - \frac{1}{2}\|s_I^-(k)\|^2 + e_C^T(k)f_C(k) + e_R^T(k)f_R(k) = 0$$  \hspace{1cm} (21)

Furthermore, during the interval $k$, we have to consider a constant state $x(k)$ corresponding to the continuous time state $x(t)$. This implies that during the interval $k$, the dissipated energy will be equal to $T\int f_R^T(k)R(x(k))f_R(k)$ and the supplied energy will be equal to $-T(\frac{1}{2}\|s_I^+(k)\|^2 - \frac{1}{2}\|s_I^-(k)\|^2)$. In order to be consistent with the energy flows, and as a consequence conserve passivity, we need therefore a jump in internal energy $\Delta H(k)$ from instant $kT$ to instant $(k+1)T$ such that:

$$\Delta H(k) = -Tf_R^T(k)R(x(k))f_R(k) - T(\frac{1}{2}\|s_I^+(k)\|^2 - \frac{1}{2}\|s_I^-(k)\|^2)$$

This implies that the new discrete state $x(k+1)$ should belong to an energetic level such that:

$$H(x(k+1)) = H(x(k)) + \Delta H(k)$$

We can indicate the set of possible energetically consistent states, which can be found solving the previous equation in $x(k+1)$, as

$$I_{k+1} := \{ x \in \mathcal{X} \ s.t. \ H(x) = H(x(k)) + \Delta H(k) \}.$$  

Furthermore, from the discrete equivalent of Eq.(15), we have that:

$$f_C(k) = S_Rs_I^+(k) + S_is_C(k)$$  \hspace{1cm} (22)

and therefore, for consistency with the continuous dynamics in which $f_C(t) = \dot{x}(t)$, the next state $x(k+1)$ should be such that:

$$f_C(k) = \lim_{T \to 0} \frac{x(k+1) - x(k)}{T}$$  \hspace{1cm} (23)

The set $I_{k+1}$ can be either empty or have more solutions. These two situation are treated exactly in the same way as in [7].

As a summary of the procedure just outlined, we hereafter algorithmically explain the way the discrete system can be integrated

1) Given an initial state $x(k)$, we set $e_C(k) = \frac{\partial H}{\partial x}(x(k))$.
2) Using the value of the system input $s_I^+(k)$ and the previously calculated $e_C(k)$, we can calculate $s_I^-(k)$,

the output of the interaction port, and $f_C(k)$ using the discrete representation of Eq.(15)
3) $f_C(k)$ is then used to calculate the next state $x(k+1)$ using the procedure explained in this subsection.

In Fig.2 is represented the general scheme for an intrinsically passive port-Hamiltonian based haptic interface.

![Fig. 2. The final scheme](image-url)

Since Proposition 1 holds, the discrete system is passive (in a discrete sense) even in the output power wave is delayed because of computational delay. The interconnection between continuous and discrete domain is made through the element $SH$ which is the passive Sample & Hold described in Sec.II-C. Since the interaction of the human operator with the virtual environment will have place through power variables (i.e. effort and flow) we endowed the scheme with a coding block which is used to interface the power variables based port with the scattering based port. In case (very frequent in haptics) that the virtual environment has an impedance causality, we have that from $f_a(k)$ and $s_I^-(k)$ we compute $s_I^+(k)$ and $e_d(k)$. Notice that now the coding procedure is safe. In fact the coding/decoding procedure is, by definition, such that the energetic behavior at the port $(s_I^-, s_I^+)$ and the one at the port $(e_d, f_d)$ are exactly the same (there is energetic synchronism); since Proposition 1 holds we have that the behavior at the port $(s_I^-, s_I^+)$ is passive and therefore we will have a passive behavior at the port $(e_d, f_d)$ independently of any computational delay.

The proposed scheme provides a passive haptics scheme for delayed virtual environment without any extra layer to dissipate extra energy produced by the delay. The scheme is based on natural and intuitive energetic consideration and it has the advantage that transparency of the virtual environment is affected only by the unavoidable dynamical effect associated to the delay and it is not perturbed by any extra dynamics.

**Remark 1:** In case there is no computational delay on the output, we have that using either power variables or scattering waves leads to the same behavior of the system. The scheme in Fig.2 can be, therefore, considered a generalization of the scheme proposed in [7].

**Remark 2:** Since the scheme proposed in [7] is passive independently of the sample period, one could think to avoid the delayed output problem by simply increasing the sample period. This is true but performances of the system are affected by the sample period used: the higher is the
sampling rate, the more realistic will be the simulation. In order to have a performant haptic interface, therefore, it is necessary to keep the sample period as small as possible taking into account, in the implementation, possible computational delays.

V. SIMULATIONS

The aim of this section is to provide some simulations in order to validate our results. Consider a simple mass, with an initial state $p = 1 \, mKg/sec$, connected with a discrete spring. The spring is implemented with the strategy depicted in [7] and the interconnection between continuous and discrete time is made by the passive Sample & Hold algorithm proposed in [7]. The sample period is $T = 10 \, ms$ and there is a computational delay $\delta = 0.05 \, sec$.

We can see that using effort/flow representation for the interaction power port, passivity is lost and unstable behavior is introduced. When using scattering representation, passivity is preserved and a stable behavior is obtained. The shift with respect to the non delayed behavior is due to the dynamical effect deriving from the output delay. The simulation results are reported in Fig.3. The last simulation is very common in haptics literature and it implements a virtual wall. The operator pushes the haptic device (a mass) with a constant force and, at $x = 0.3 \, m$ the system meets a virtual wall (implemented with a very stiff spring and a damper). We can see in Fig.4 that, even if the operator keeps on pushing, the mass is still at $x = 0.3 \, m$ because of the action of the virtual wall. The virtual wall is simulated with $T = 0.01 \, sec$ and there is a computational delay of 3 sample periods.

VI. CONCLUSIONS AND FUTURE WORK

In this paper we addressed the problem of delayed virtual environments in haptics using a port-Hamiltonian approach. We have shown that using scattering waves as input and output of a port-Hamiltonian system, preserves a passive behavior even in case of output delay. We then provided a passivity preserving discretization algorithm for scattered port-Hamiltonian systems in order to be able to build a scheme for the implementation of an haptic interface which is passive independently of the sample period and of the computational delay. Future work will be devoted to the implementation of the scheme on an experimental setup.

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VII. REFERENCES